1.0

1.1

1.25

4.5
5.0
5.6
6.3

2.8

3.2

3.6

4.0

2.5

2.2

2.0

1.4

1.8

1.6

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MULTIPLIER OFFSET VOLTAGES IN ADAPTIVE ARRAYS

R.T. Compton, Jr.

The Ohio State University

ElectroScience Laboratory

Department of Electrical Engineering
Columbus, Ohio 43212

QUARTERLY REPORT 3433-1

October 1972

Contract N00019-72-C-0184

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MULTIPLIER OFFSET VOLTAGES IN ADAPTIVE ARRAYS

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ABSTRACT

This report examines the effects of multiplier offset voltages
in adaptive arrays. Multiplier offset voltages arise when active
circuits are used to implement the error-by-signal multipliers re-
quired in an array based on the LMS algorithm. These offset voltages
are known from experimental work to have a strong effect on array
performance.

It is first shown how multiplier offset voltages may be included
in the differential equations for the array weights. Then their effect
on weight behavior is studied. It is found that the offset voltages
affect the final values of the weights, but not the time constants.
Furthermore, the effect they have is influenced by the amount of
element noise in the array. An adequate amount of noise is necessary
to minimize weight errors due to offset voltages.

An example is treated to show the effect of offset voltages on
the final array weights and the output SNR. With offset voltages
present, it is found that there is a maximum SNR that can be obtained
from the array. A specific input SNR is required to obtain this maxi-
mum output SNR.

Finally, it is shown that a finite operating range for the weights
places a further restriction on the acceptable values of offset volt-
ages and noise.

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I. INTRODUCTION

Adaptive arrays have been under investigation for several years. Widrow, et.al.[1], suggested the LMS algorithm and did computer simulations of arrays based on this concept. Shor[2] and Applebaum[3] have also discussed closely related concepts. An early experimental adaptive array was built by Riegler and Compton[4]. Recently, a 4-element adaptive array was constructed and used to perform extensive pattern measurements with elements on an irregularly shaped surface [5,6]. Adaptive array techniques applicable to spread spectrum communication systems are also under study[7,8,9,10,11].

Adaptive arrays based on the LMS algorithm require the signal on each channel of the array to be multiplied by the error signal (the difference between the array output and a reference signal). It has been found experimentally that the design of these multipliers is a critical factor in obtaining good performance from the array. Specifically, the problem centers around the presence of small d-c offset voltages at the outputs of the multipliers. These offset voltages are unrelated to the signals; they arise because the multipliers are implemented with active circuit devices. (For example, in an array under study at Ohio State, transconductance multipliers have been used. The offset voltages are due to imperfect balancing in the devices and the associated circuits.) Since the output from each multiplier goes directly into an integrator that controls an array weight, the offset voltages cause the array weights to be in error and thus can have a strong effect on array performance. Furthermore, it has been found that the effect of the offsets depends on the amount of noise present in the array. A certain amount of noise seems to be necessary to counter-act the offsets.

The purpose of this report is to study these effects from a theoretical standpoint. In Section II, we show how the offset voltages may be included in the differential equations for the weights, and what their effect on the weight behavior is. It is found that the form of the weight transients depends on whether the array is underconstrained or not. In Section III, it is shown that the array is underconstrained only when the number of signals incident is fewer than the number of elements and when there is no noise. In Section IV, a 2-element array with one signal incident and with noise is analyzed to show the effects of the offset voltages. Finally, in Section V, we discuss the fact that the array weights have only a finite operating range, and show how this limitation imposes a further restraint on the acceptable values of the offsets and noise.

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II. THE ANALYTICAL SOLUTION

The general configuration of an N-element adaptive array is shown in Fig. 1. The signal from each element, $y_i(t)$, is split into in-phase and quadrature components $x_i(t)$. Each $x_i(t)$ is weighted by a real coefficient w_i and then summed to produce the array output $s(t)$. The difference between the array output and a reference signal $R(t)$, which is called the error signal $\epsilon(t)$, forms the input to a feedback system that adjusts the w_i.
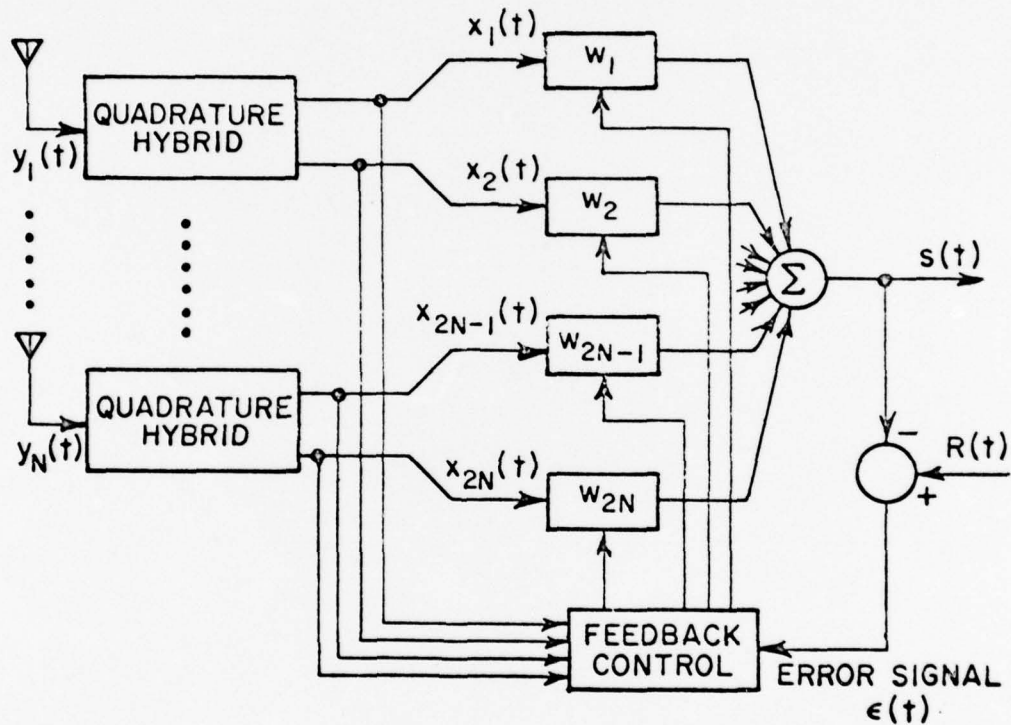

Fig. 1. Adaptive array structure.

The feedback system is based on the so-called LMS algorithm[1,4]. Each weight is adjusted according to

$$(1) \qquad \frac{dw_i}{dt} = - k \, \nabla_{w_i} \overline{[\epsilon^2(t)]} \quad ,$$

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where $\nabla_{w_i}[\overline{\varepsilon^2(t)}]$ denotes the i-th component of the gradient of the mean-square error signal $\overline{\varepsilon^2(t)}$. Since the error signal is given by

$$(2) \qquad \varepsilon(t) = R(t) - \sum_{i=1}^{2N} w_i \, x_i(t)$$

the mean-square error is

$$(3)* \qquad \overline{\varepsilon^2(t)} = \overline{R^2(t)} - 2 \sum_{i=1}^{2N} w_i \, \overline{x_i(t)R(t)}$$

$$+ \sum_{i=1}^{2N} \sum_{j=1}^{2N} w_i w_j \, \overline{x_i(t)x_j(t)} \qquad .$$

Differentiating Eq. (3) with respect to w_i yields

$$(4) \qquad \nabla_{w_i}[\overline{\varepsilon^2(t)}] = \frac{\partial \overline{\varepsilon^2(t)}}{\partial w_i} = - 2 \, \overline{x_i(t)\varepsilon(t)} \qquad ,$$

so Eq. (1) becomes

$$(5) \qquad \frac{dw_i}{dt} = 2k \, \overline{\varepsilon(t)x_i(t)} \qquad .$$

Equation (5) leads to the feedback loop structure shown in Fig. 2. This feedback structure has been the basis for much of the recent work in adaptive arrays[5,6,7,8,9].

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*The overbar here represents the action of a low-pass filter, as discussed in Reference 10, page 5. For the present discussion, it is also the same as an infinite time average.
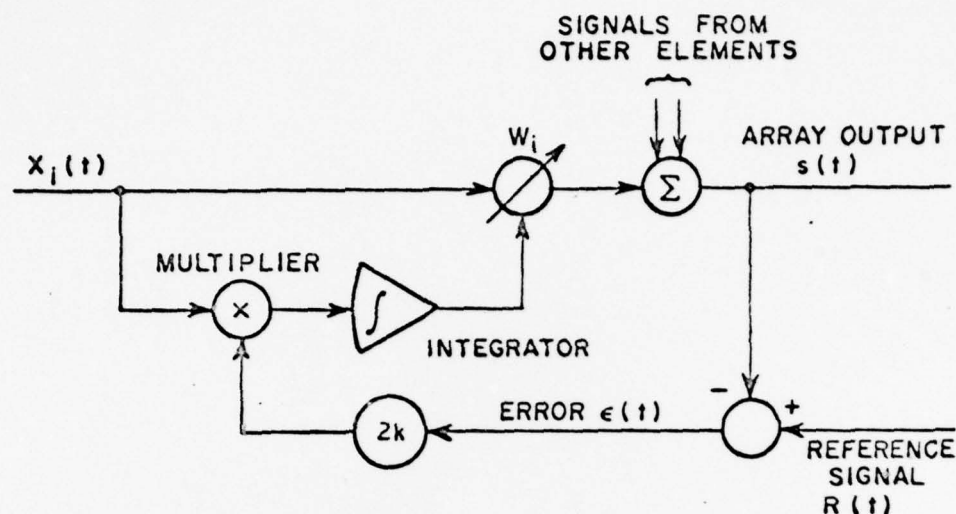
3

Fig. 2. Adaptive array feedback loop.

When the feedback system shown in Fig. 2 is implemented, it is found that the error-by-signal multiplier is the most critical part of the design problem. Ideally, this multiplier should generate the product $\overline{\epsilon(t)x_i(t)}$. In practice, a multiplier using active circuit devices is found to be subject to leakage effects, nonlinearities, and circuit imbalances. As a result, the multiplier output may contain terms such as the following:

$$\text{Multiplier Output} = \delta_i + \overline{\epsilon(t)x_i(t)} + c_1\,x_i(t) + c_2\,\epsilon(t)$$

$$+ c_3\,\overline{x_i^2(t)} + c_4\,\overline{\epsilon^2(t)} + c_5\,\overline{x_i^2(t)\epsilon(t)} + \cdots$$

The term $\overline{\epsilon(t)x_i(t)}$ is the desired output from the multiplier, and in a well-designed circuit is the dominant term. The term δ_i is a small d-c voltage unrelated to the signals $x_i(t)$ or $\epsilon(t)$. We refer to δ_i as a Multiplier Offset Voltage. We will see below that this term can have a strong effect on array performance. The physical mechanism responsible for δ_i depends on the type of multiplier used. A trans-conductance multiplier is one type that has been used[5,7], for example, and in this case the offset voltages are due to inadequate balancing of the transistor cells.

The terms $c_1x_i(t)$ and $c_2\epsilon(t)$ represent leakage of the input sig-nals into the multiplier output. Since the output of the multiplier goes directly into an integrator, only those multiplier output components

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centered around zero frequency are important. In adaptive arrays for radio communications, the signals $x_i(t)$ and $\varepsilon(t)$ are bandlimited signals at a nonzero carrier frequency and thus their effect on the multiplier output can be ignored.*

Terms such as $c_3 x_i{}^2(t)$, $c_4 \varepsilon^2(t)$, $c_5 x_i{}^2(t)\varepsilon(t)$, and similar higher order terms, result from nonlinearities in the circuit devices used in the multiplier. Although $c_3 x_i{}^2(t)$ has a d-c component, this term has been found to be negligible in practice. If it were not negligible, however, it could be lumped together with δ_i for purposes of the present analysis. The term $c_4 \varepsilon^2(t)$ also has a d-c component, but since the error signal is small when the array is in steady-state, this term has no effect on the steady-state performance. Higher-order terms such as $c_5 x_i{}^2(t)\varepsilon(t)$ have been found to be negligible in practice.

Thus, we model the error-by-signal multiplier by the equation

(6)   Multiplier Output = $\delta_i + \overline{\varepsilon(t)x_i(t)}$.

Our purpose in this section of the report is to show how the effects of the offset voltages δ_i may be analyzed.

We begin by examining the differential equations for the weights. If the output of each error-by-signal multiplier in the array is of the form in Eq. (6), the array weights satisfy the differential equations

(7) $\dfrac{dw_i}{dt} = 2k \left[\varepsilon(t)x_i(t) + \delta_i \right]$, $1 \le i \le 2N$.

When Eq. (2) is used to substitute for $\varepsilon(t)$ in Eq. (7), and all terms involving w_i are collected on the left, it is found that the weights satisfy the system of differential equations

(8) $\dfrac{dw_i}{dt} + 2k \displaystyle\sum_{j=1}^{2N} \overline{\left[x_i(t)x_j(t) \right]} w_j = 2k \left[\overline{R(t)x_i(t)} + \delta_i \right]$.

---

*In adaptive arrays for sonar or seismic applications where baseband signals are processed however, these terms could be important.

We define the matrices

$$(9) \qquad \Phi = \begin{pmatrix} \overline{x_1(t)x_1(t)} & \overline{x_1(t)x_2(t)} & - - - - \\ \overline{x_2(t)x_1(t)} & & \\ \vdots & & \end{pmatrix} \quad ,$$

$$(10) \qquad S = \begin{pmatrix} \overline{R(t)x_1(t)} \\ \overline{R(t)x_2(t)} \\ \vdots \\ \vdots \end{pmatrix} \quad ,$$

$$(11) \qquad W = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \end{pmatrix} \quad ,$$

and

$$(12) \qquad \Delta = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \vdots \end{pmatrix} \quad .$$

Then Eq. (8) may be written in matrix form as

$$(13) \qquad \frac{dw}{dt} + 2k\Phi w = 2k[S + \Delta] \quad .$$

Clearly Δ (which we call the "offset voltage vector") plays the same role in the differential equations for the weights as does the vector S, the correlation between the reference signal and the array signals.

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Let us consider the transient response of Eq. (13). First, we make a rotation of coordinates into the principal axes of ϕ. Let

(14) $\qquad w = Rn$,

where R is a 2N x 2N orthogonal coordinate rotation matrix,

(15) $\qquad R = \begin{pmatrix} r_{11} & r_{12} & - - - \\ r_{21} & & \\ \vdots & & \end{pmatrix}$,

and

(16) $\qquad n = \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ \vdots \end{pmatrix}$

represents a new system of coordinates for the weights. By substituting Eq. (14) into Eq. (13) and multiplying on the left by R^{-1}, Eq. (13) becomes

(17) $\qquad \dfrac{dn}{dt} + 2k \left[R^{-1} \phi R \right] n = 2k \, R^{-1} \left[S + \Delta \right]$.

If R is chosen so $R^{-1} \phi R$ is diagonal,

(18) $\qquad R^{-1} \phi R = \Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 & - - - \\ 0 & \lambda_2 & 0 & \\ 0 & 0 & \lambda_3 & \\ \vdots & & & \ddots \\ \vdots & & & \end{pmatrix}$,

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then the components of η lie along the principle axes of ϕ and the system of equations (17) is uncoupled. We define

$$(19) \qquad P = R^{-1}S = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \end{pmatrix} \quad ,$$

and

$$(20) \qquad Q = R^{-1}\Delta = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \end{pmatrix} \quad ,$$

and Eq. (17) becomes simply

$$(21) \qquad \frac{d\eta}{dt} + 2k\Lambda\eta = 2k\,[P + Q] \quad .$$

We refer to the components of η as the "normal weights" in the array.

The form of the general solution to Eq. (21) depends on the matrix ϕ. Since ϕ is real and symmetric, its eigenvalues are necessarily real. Furthermore, ϕ is non-negative definite. To see this, we note that the mean-square error in Eq. (3) may be written in matrix form as

$$(22) \qquad \overline{\varepsilon^2(t)} = \overline{R^2(t)} - 2w^T S + w^T \phi w \quad .$$

(Superscript T denotes the transpose.) This may be rearranged[10] into the form

$$(23) \qquad \overline{\varepsilon^2(t)} = \overline{\varepsilon_{min}^2} + (w - w_{min})^T \phi\, (w - w_{min}) \quad ,$$

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where

$$(24) \qquad \varepsilon_{min}^2 = R^2(t) - S^T \phi^{-1} S \qquad ,$$

and

$$(25) \qquad w_{min} = \phi^{-1} S \qquad .$$

($\overline{\varepsilon_{min}^2}$ is the minimum value $\overline{\varepsilon^2(t)}$ for any weight w, and w_{min} is the value of w yielding $\overline{\varepsilon^2(t)} = \overline{\varepsilon_{min}^2}$.) Since $\overline{\varepsilon^2(t)}$ is the square of a real quantity, it cannot be negative, and thus the eigenvalues of ϕ cannot be negative. Otherwise, large enough values of $w - w_{min}$ would yield negative $\overline{\varepsilon^2(t)}$.

Since none of the eigenvalues of ϕ can be negative, the solutions to Eq. (21) will not contain any exponentially growing terms. Some of the eigenvalues can be zero (ϕ can be singular), however, when there is no noise in the array. The solutions to Eq. (21) will be different with zero eigenvalues (ϕ singular) than with all nonzero eigenvalues (ϕ nonsingular).

When some of the eigenvalues are zero, the system in Eq. (21) will contain two types of differential equations. The n_i associated with nonzero λ_i will satisfy equations of the type

$$(26) \qquad \frac{dn_i}{dt} + 2k\lambda_i n_i = 2k(p_i + q_i) \qquad .$$

These have solutions of the form

$$(27) \qquad n_i(t) = A_i e^{-2k\lambda_i t} + \frac{p_i + q_i}{\lambda_i} \qquad .$$

The constants of integration A_i are found from the initial values of $n_i(t)$ at t=0:

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$$(28) \qquad A_i = n_i(0) - \frac{p_i + q_i}{\lambda_i} \qquad .$$

For these $n_i(t)$, the effect of the offset voltages δ_i (which are transformed into the q_i) is to alter the steady-state solutions of the weights, given by

$$(29) \qquad n_i(\infty) = \frac{p_i + q_i}{\lambda_i} \qquad .$$

The n_i associated with zero eigenvalues, on the other hand, satisfy equations of the form

$$(30) \qquad \frac{dn_i}{dt} = 2k(p_i + q_i) \qquad ,$$

for which the solutions are simply

$$(31) \qquad n_i(t) = A_i + 2k(p_i + q_i)t \qquad .$$

Again, the constants A_i are determined from the initial values of the $n_i(t)$:

$$(32) \qquad A_i = n_i(0) \qquad .$$

The effect of the q_i terms here is to alter the slope of the ramp functions in $n_i(t)$.

Thus, the offset voltages affect the final values of the normal weights associated with nonzero eigenvalues, and affect the slopes of the ramp response terms for normal weights associated with zero eigenvalues. We note that if any of the eigenvalues are zero, the array weights never reach a steady-state condition, because the ramp functions continue indefinitely. Since w is related to n by Eq. (14), we see that in general each of the weights w_i will contain both damped exponential terms and linear ramp function terms.

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For the case where all of the λ_i are nonzero (ϕ nonsingular) all of the normal weights n_i will satisfy Eq. (26). The ramp response terms will not be present in the solution for the weights.

When ϕ is nonsingular, the weights approach a steady-state solution w_{ss}, which may be found directly from Eq. (13):

$$(33) \qquad w_{ss} = \phi^{-1}[S + \Delta] \qquad .$$

(Since ϕ is nonsingular, its inverse exists.) We note that in the absence of multiplier offset voltages, the steady-state weight vector would be $w_{min} = \phi^{-1}S$, as given in Eq. (25). Hence the offset voltages shift the weights from their optimum point by an amount $\phi^{-1}\Delta$. Substituting

$$(34) \qquad w - w_{min} = \phi^{-1}\Delta$$

into Eq. (23) shows that the steady-state mean-square error will be

$$(35) \qquad \overline{\varepsilon_{ss}^2(t)} = \overline{\varepsilon_{min}^2} + \Delta^T \phi^{-1} \Delta$$

i.e., the offset voltages increase the mean-square error by an amount $\Delta^T \phi^{-1} \Delta$ over its value with the optimum weights. Since the steady-state mean-square error signal is closely related to interference null depths and output signal-to-noise ratio (SNR), the effect of this term is to lower the interference rejection of the array or to lower the output SNR. Whether this change is significant or not depends on the values of Δ and ϕ, and also the value of ε_{min}^2. We consider an example in Section IV.

It is interesting that the steady-state weights in Eq. (33) result in zero output voltages from the error-by-signal multipliers. To see this, we note that the steady-state error signal is

$$(36) \qquad \varepsilon_{ss}(t) = R(t) - X^T w_{ss}$$

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where

$$(37) \qquad X = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \end{pmatrix} \qquad .$$

Since the output of the i-th multiplier is $\overline{\epsilon(t)x_i(t)} + \delta_i$, the steady-state output of the multipliers, expressed in vector form, is

$$(38) \qquad \overline{X\epsilon_{ss}(t)} + \Delta = \overline{XR(t)} - \overline{XX^T}\, w_{ss} + \Delta = S - \Phi\, w_{ss} + \Delta \qquad .$$

But substituting for w_{ss} with Eq. (33) yields

$$(39) \qquad \overline{X\epsilon_{ss}(t)} + \Delta = S - \Phi\Phi^{-1}[S + \Delta] + \Delta = 0 \qquad .$$

All the multiplier outputs are zero in the steady-state.*

　　　Physically, the steady-state error signal has just the right value that the product $\epsilon(t)x_i(t)$ at the output of each multiplier cancels the offset voltage δ_i. This means that the residual error signal, $\epsilon_{ss}(t)$, is larger than its minimum possible value, by an amount determined by the offset voltages. Of course, the larger this residual error signal, the larger the amount of interference in the array output, and the less the interference protection of the array.

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*This analysis treats only average values. In actual fact, when there is noise present the multiplier output is the product of two random processes. Thus its spectrum contains an impulse function at d-c plus continuous frequency components over a finite band. The above analysis considers only the d-c term.

Thus, to summarize, we find that when ϕ is singular, the weights do not have a steady-state solution. They increase without limit, owing to ramp functions in the solution. When ϕ is nonsingular, the weights have a steady-state solution given by Eq. (33). The solution causes the mean-square error signal to be larger than its optimum value by an amount $\Delta^T \phi^{-1} \Delta$. The residual error signal is just sufficient for the error-by-signal product at the output of each multiplier to cancel the offset voltage.

III. SINGULARITY OF ϕ

Having shown that the form of the solution to Eq. (13) depends on whether ϕ is singular or not, we next discuss the conditions under which ϕ is singular.

For ϕ to be singular, two conditions are necessary. First, there must be no element noise in the signals $x_i(t)$, and second, the array must be underconstrained.

Consider first the effect of signals incident on the array. It has been shown in a previous report[12] that in the absence of element noise, the rank of ϕ is equal to twice the number of signals incident on the array. In an N-element array, ϕ is of order 2N x 2N, and thus the rank of ϕ will be less than 2N whenever there are fewer than N signals incident on the array. In this case, ϕ is singular and we say the array is underconstrained.

Next consider the effect of element noise. By "element noise", we mean noise due to RF components (e.g., mixers) behind each element of the array. This type of noise is incoherent from one element to the next. Element noise does not refer to a directional noise signal received by the array; such noise would be highly correlated between elements.

When element noise is present, we have

$$(40) \qquad x_i(t) = n_i(t) + s_i(t)$$

where $n_i(t)$ is the noise component and $s_i(t)$ is the received signal component of $x_i(t)$. When this $x_i(t)$ is substituted into Eq. (9), ϕ is found to be

$$(41) \qquad \phi = \phi_{s+n} = \sigma_n^2 I + \phi_s$$

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where we use ϕ_{s+n} to denote ϕ when both signal and noise are present and ϕ_s when only signal is present. σ_n^2 denotes the mean-square value of $n_i(t)$:

$$(42) \qquad \overline{n_i^2(t)} = \sigma_n^2$$

(we assume all $n_i(t)$ have the same mean-square value), and I denotes the identity matrix. To derive Eq. (41), we have made use of the assumption that

$$(43) \qquad \overline{n_i(t)n_j(t)} = 0 \qquad \text{for } i \neq j \quad ,$$

and

$$(44) \qquad \overline{n_i(t)s_j(t)} = 0 \qquad \text{for all } i,j \quad .$$

Equation (43) follows because the noise signals are uncorrelated between elements and also two $n_i(t)$ associated with the same element have carriers that are in quadrature. Equation (44) follows because the noise and signal components are independent.

ϕ_s is due to signal alone. It is singular if the array is underconstrained, as mentioned above. It is also non-negative definite -- none of its eigenvalues can be negative. Since the matrix $\sigma_n^2 I$ is unaffected by a transformation of the type $R^{-1}(\sigma_n^2 I)R = \sigma_n^2 I$, the same orthogonal matrix that diagonalizes ϕ_s will diagonalize ϕ_{s+n}. Hence each eigenvalue of ϕ_{s+n} must be equal to σ_n^2 plus the corresponding eigenvalue of ϕ_s. Since none of the eigenvalues of ϕ_s can be negative (and of course $\sigma_n^2 > 0$), ϕ_{s+n} cannot be singular.

Thus, we have shown that ϕ cannot be singular except when there is no element noise and the array is underconstrained.

IV.   AN EXAMPLE

Now let us consider a simple example. Suppose we have a 2 element array, as shown in Fig. 3. Suppose there is one CW signal
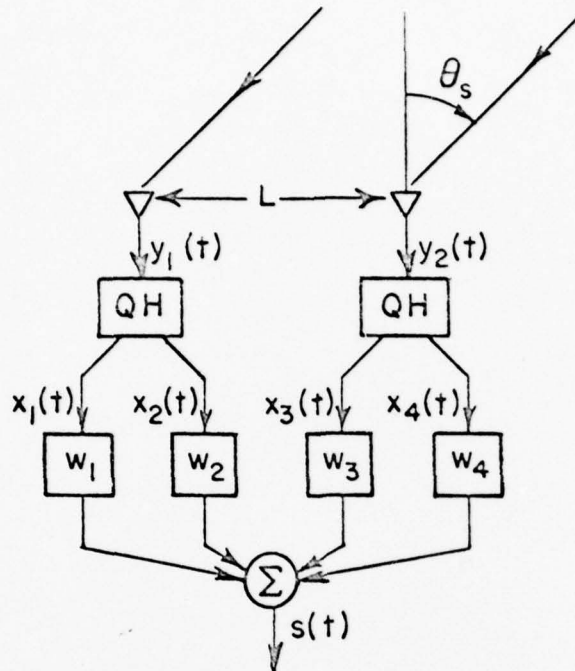
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Fig. 3. A two-element array.

incident on the array from angle θ_1 with respect to broadside. Initially, we will assume there is no noise. The element signals are given by*

$$(45) \qquad y_1(t) = \sqrt{2} \, a \, \cos[\omega_1 t - \phi_1]$$

$$(46) \qquad y_2(t) = \sqrt{2} \, a \, \cos[\omega_1 t]$$

where

*The factor $\sqrt{2}$ is included to make the in-phase and quadrature signals in Eqs. (48) - (51) have unit amplitude.

$$(47) \qquad \phi_1 = \frac{2\pi L}{\lambda_0} \sin\theta_1$$

(L is the element spacing and λ_0 is the free-space wavelength.) The in-phase and quadrature signals are

$$(48) \qquad x_1(t) = s_1(t) = a \cos[\omega_1 t - \phi_1] \quad ,$$

$$(49) \qquad x_2(t) = s_2(t) = a \sin[\omega_1 t - \phi_1] \quad ,$$

$$(50) \qquad x_3(t) = s_3(t) = a \cos[\omega_1 t] \quad ,$$

and

$$(51) \qquad x_4(t) = s_4(t) = a \sin[\omega_1 t] \quad .$$

Substituting these into Eq. (9) yields for ϕ:

$$(52) \qquad \phi = \phi_s = \begin{pmatrix} \frac{a^2}{2} & 0 & \frac{a^2}{2}\cos\phi_1 & \frac{a^2}{2}\sin\phi_1 \\[2ex] 0 & \frac{a^2}{2} & -\frac{a^2}{2}\sin\phi_1 & \frac{a^2}{2}\cos\phi_1 \\[2ex] \frac{a^2}{2}\cos\phi_1 & -\frac{a^2}{2}\sin\phi_1 & \frac{a^2}{2} & 0 \\[2ex] \frac{a^2}{2}\sin\phi_1 & \frac{a^2}{2}\cos\phi_1 & 0 & \frac{a^2}{2} \end{pmatrix} \quad .$$

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(The subscript s indicates that this matrix applies when only signal is present.) This matrix has been studied previously in Reference 10. An orthogonal coordinate rotation matrix that diagonalizes ϕ_s is given by

$$(53) \qquad R = \frac{1}{\sqrt{2}} \begin{pmatrix} -\cos\phi_1 & -\sin\phi_1 & \cos\phi_1 & \sin\phi_1 \\ \sin\phi_1 & -\cos\phi_1 & -\sin\phi_1 & \cos\phi_1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad .$$

The product $R^{-1}\phi_s R$ is found to be

$$(54)^* \qquad R^{-1}\phi_s R = \Lambda_s = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{pmatrix}$$

so the eigenvalues of ϕ_s are 0, 0, a^2, and a^2. Since ϕ_s has two zero eigenvalues and two nonzero eigenvalues, it is of rank 2 (twice the number of signals).

Suppose furthermore that the reference signal is given by

$$(55) \qquad R(t) = \cos\omega_1 t \quad .$$

In other words, R(t) is a signal coherent with the incoming signal. (The incoming signal is "desired".) Then the vector S in Eq. (10) is found to be

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*Since R is an orthogonal matrix, $R^T = R^{-1}$.

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$$(56) \qquad S = \begin{pmatrix} \dfrac{a}{2}\cos\phi_1 \\[2mm] -\dfrac{a}{2}\sin\phi_1 \\[2mm] \dfrac{a}{2} \\[2mm] 0 \end{pmatrix} \quad .$$

The weights w_i satisfy the system in Eq. (13):

$$(57) \qquad \frac{dw}{dt} + 2k\Phi_s w = 2k[S + \Delta] \quad .$$

Making the coordinate rotation in Eq. (14) yields the equivalent system of Eq. (21)

$$(58) \qquad \frac{d\eta}{dt} + 2k\Lambda_s \eta = 2kR^T[S + \Delta] = 2k[P + Q] \quad ,$$

or

$$(59) \qquad \frac{d}{dt}\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix} + 2k\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix} = 2k\begin{pmatrix} p_1 + q_1 \\ p_2 + q_2 \\ p_3 + q_3 \\ p_4 + q_4 \end{pmatrix} \quad .$$

The vector $P = R^T S$ is found to be

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$$
(60) \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\cos\phi_1 & \sin\phi_1 & 1 & 0 \\ -\sin\phi_1 & -\cos\phi_1 & 0 & 1 \\ \cos\phi_1 & -\sin\phi_1 & 1 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{a}{2}\cos\phi_1 \\ -\frac{a}{2}\sin\phi_1 \\ \frac{a}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ a/\sqrt{2} \\ 0 \end{pmatrix} \quad ,
$$

so the system in Eq. (59) yields the four uncoupled equations,

$$
(61) \qquad \frac{d\eta_1}{dt} = 2kq_1 \qquad ,
$$

$$
(62) \qquad \frac{d\eta_2}{dt} = 2kq_2 \qquad ,
$$

$$
(63) \qquad \frac{d\eta_3}{dt} + 2ka^2\eta_3 = 2k\left[\frac{a}{\sqrt{2}} + q_3\right] \qquad ,
$$

and

$$
(64) \qquad \frac{d\eta_4}{dt} + 2ka^2\eta_4 = 2kq_4 \qquad .
$$

The solutions are

$$
(65) \qquad \eta_1(t) = \eta_1(0) + 2kq_1 t \qquad ,
$$

$$
(66) \qquad \eta_2(t) = \eta_2(0) + 2kq_2 t \qquad ,
$$

$$
(67) \qquad \eta_3(t) = \left[\eta_3(0) - \frac{1}{a^2}\left(\frac{a}{\sqrt{2}} + q_3\right)\right] e^{-2ka^2 t} + \frac{1}{a^2}\left[\frac{a}{\sqrt{2}} + q_3\right] \qquad ,
$$

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and

$$(68) \qquad n_4(t) = \left[n_4(0) - \frac{q_4}{a^2} \right] e^{-2ka^2 t} + \frac{q_4}{a^2} \qquad ,$$

where the terms $n_i(0)$ denote the initial values of $n_i(t)$ at t=0. These initial values are found from the initial values of the w_i, according to the inverse relation to Eq. (14):

$$(69) \qquad n = R^T w \qquad .$$

In order to be specific, let us assume that the initial values of the w_i are

$$(70) \qquad \begin{pmatrix} w_1(0) \\ w_2(0) \\ w_3(0) \\ w_4(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad .$$

Then we find from Eq. (69)

$$(71) \qquad \begin{pmatrix} n_1(0) \\ n_2(0) \\ n_3(0) \\ n_4(0) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\cos\phi_1 \\ -\sin\phi_1 \\ \cos\phi_1 \\ \sin\phi_1 \end{pmatrix} \qquad .$$

Thus, Eqs. (65) - (68) become

$$(72) \qquad n_1(t) = - \frac{\cos\phi_1}{\sqrt{2}} + 2kq_1 t \qquad ,$$

$$(73) \qquad \eta_2(t) = -\frac{\sin\phi_1}{\sqrt{2}} + 2kq_2 t \qquad ,$$

$$(74) \qquad \eta_3(t) = \left[\frac{\cos\phi_1}{\sqrt{2}} - \frac{1}{a^2}\left(\frac{a}{\sqrt{2}} + q_3\right)\right] e^{-2ka^2 t} + \frac{1}{a^2}\left(\frac{a}{\sqrt{2}} + q_3\right) \quad ,$$

and

$$(75) \qquad \eta_4(t) = \left[\frac{\sin\phi_1}{\sqrt{2}} - \frac{q_4}{a^2}\right] e^{-2ka^2 t} + \frac{q_4}{a^2} \qquad .$$

The array weights w_i can now be found by applying the transformation of Eq. (14) again:

$$(76) \qquad \begin{pmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\cos\phi_1 & -\sin\phi_1 & \cos\phi_1 & \sin\phi_1 \\ \sin\phi_1 & -\cos\phi_1 & -\sin\phi_1 & \cos\phi_1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1(t) \\ \eta_2(t) \\ \eta_3(t) \\ \eta_4(t) \end{pmatrix} .$$

The result is

$$(77) \qquad w_1(t) = \frac{1}{2} + K(\delta_1 - \cos\phi_1\,\delta_3 - \sin\phi_1\,\delta_4) t$$

$$+ \left[\frac{1}{2} - \frac{\cos\phi_1}{2a} - \frac{1}{2a^2}(\delta_1 + \cos\phi_1 \delta_3 + \sin\phi_1 \delta_4)\right] e^{-2ka^2 t}$$

$$+ \frac{\cos\phi_1}{2a} + \frac{1}{2a^2}(\delta_1 + \cos\phi_1\delta_3 + \sin\phi_1\delta_4) \quad ,$$

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(78) $\quad w_2(t) = K(\delta_2 + \sin\phi_1\delta_3 - \cos\phi_1\delta_4)t$

$$+ \left[\frac{\sin\phi_1}{2a} + \frac{1}{2a^2}(-\delta_2 + \sin\phi_1\delta_3 - \cos\phi_1\delta_4)\right]e^{-2ka^2t}$$

$$- \frac{\sin\phi_1}{2a} - \frac{1}{2a^2}(-\delta_2 + \sin\phi_1\delta_3 - \cos\phi_1\delta_4) \quad ,$$

(79) $\quad w_3(t) = -\frac{\cos\phi_1}{2} + K(-\cos\phi_1\delta_1 + \sin\phi_1\delta_2 + \delta_3)\ t$

$$+ \left[\frac{\cos\phi_1}{2} - \frac{1}{2a^2}(a + \cos\phi_1\delta_1 - \sin\phi_1\delta_2 + \delta_3)\right]e^{-2ka^2t}$$

$$+ \frac{1}{2a^2}(a + \cos\phi_1\delta_1 - \sin\phi_1\delta_2 + \delta_3) \quad ,$$

and

(80) $\quad w_4(t) = -\frac{\sin\phi_1}{2} + K(-\sin\phi_1\delta_1 - \cos\phi_1\delta_2 + \delta_4)t$

$$+ \left[\frac{\sin\phi_1}{2} - \frac{1}{2a^2}(\sin\phi_1\delta_1 + \cos\phi_1\delta_2 + \delta_4)\right]e^{-2ka^2t}$$

$$+ \frac{1}{2a^2}(\sin\phi_1\delta_1 + \cos\phi_1\delta_2 + \delta_4) \quad ,$$

where we have used Eq. (20)

(81) $\quad Q = R^T\Delta$

to substitute for the q_i in Eqs. (72) - (75).

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The effect of the multiplier offset voltages can now be seen. First, if these voltages were zero, the solution would be simply

$$(82) \qquad w_1(t) = \frac{1}{2} + \left(\frac{1}{2} - \frac{\cos\phi_1}{2a}\right) e^{-2ka^2t} + \frac{\cos\phi_1}{2a} \qquad ,$$

$$(83) \qquad w_2(t) = \frac{\sin\phi_1}{2a} (e^{-2ka^2t} - 1) \qquad ,$$

$$(84) \qquad w_3(t) = \left(\frac{\cos\phi_1}{2} - \frac{1}{2a}\right) (e^{-2ka^2t} - 1) \qquad ,$$

and

$$(85) \qquad w_4(t) = \frac{\sin\phi_1}{2} (e^{-2ka^2t} - 1) \qquad .$$

In the steady-state, after the transient has died out, the weights would have the values

$$(86) \qquad w_1(\infty) = \frac{1}{2} + \frac{\cos\phi_1}{2a} \qquad ,$$

$$(87) \qquad w_2(\infty) = - \frac{\sin\phi_1}{2a} \qquad ,$$

$$(88) \qquad w_3(\infty) = - \frac{\cos\phi_1}{2} + \frac{1}{2a} \qquad ,$$

and

$$(89) \qquad w_4(\infty) = - \frac{\sin\phi_1}{2} \qquad .$$

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The array output signal would be

$$(90) \qquad s(t) = \sum_{i=1}^{4} w_i x_i(t) = \left(\frac{1}{2} + \frac{\cos\phi_1}{2a}\right) \ a \cos(\omega_1 t - \phi_1)$$

$$+ \ \left(-\frac{\sin\phi_1}{2a}\right) \ a \sin(\omega_1 t - \phi_1) + \left(-\frac{\cos\phi_1}{2} + \frac{1}{2a}\right) a \cos\omega_1 t$$

$$-\frac{\sin\phi_1}{2} \ a \sin\omega_1 t = \cos\omega_1 t \quad .$$

This matches the reference signal in Eq. (55) exactly, so the steady-state mean-square error $\varepsilon_{ss}^2(t)$ is zero.

When multiplier offset voltages are present, each of the weights in Eqs. (77) - (80) contains a term that is linear with time. In that case, there is no steady-state solution. Each of the weights increases without limit. (In a practical array, each weight has a finite operating range over which it can vary. A weight will increase until it hits its maximum or minimum value. More will be said of this later.)

Next let us suppose that noise is present on each of the signals $x_i(t)$. Instead of Eqs. (48) - (51), we write

$$(91) \qquad x_1(t) = n_1(t) + a \cos[\omega_1 t - \phi_1]$$

$$(92) \qquad x_2(t) = n_2(t) + a \sin[\omega_1 t - \phi_1]$$

$$(93) \qquad x_3(t) = n_3(t) + a \cos[\omega_1 t]$$

$$(94) \qquad x_4(t) = n_4(t) + a \sin[\omega_1 t] \qquad .$$

The noise signals have average power σ_n^2,

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(95) $\overline{n_i^2(t)} = \sigma_n^2$

and are uncorrelated from one channel to the next,

(96) $\overline{n_i(t)n_j(t)} = 0$ for $i \neq j$.

When Eqs. (91) - (94) are substituted into Eq. (9), ϕ is found to be the same as in Eq. (52) except that an extra term $\sigma_n^2 I$ is added to the diagonal terms, as discussed in Section III.

(97) $\phi_{s+n} = \sigma_n^2 I + \phi_s$,

where I is the identity matrix. The subscript "s+n" indicates that both signal and noise are present.

Since the matrix $\sigma_n^2 I$ will be unaffected by the diagonalization transformation $R^{-1}\phi_{s+n}R$, the same matrix R as given in Eq. (53) will also diagonalize ϕ_{s+n}. The result is

(98) $R^T\phi_{s+n}R = R^T[\sigma_n^2 I+\phi_s]R = \sigma_n^2 I+\Lambda_s = \begin{pmatrix} \sigma_n^2 & 0 & 0 & 0 \\ 0 & \sigma_n^2 & 0 & 0 \\ 0 & 0 & \sigma_n^2+a^2 & 0 \\ 0 & 0 & 0 & \sigma_n^2+a^2 \end{pmatrix}$

Because of the noise terms, this matrix is no longer singular.

The differential equation for the weights, Eq. (13), becomes

(99) $\dfrac{dw}{dt} + 2k\phi_{s+n}w = 2k[S + \Delta]$.

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Applying the transformation in Eq. (14) yields Eq. (21), which now has the form,

$$(100) \qquad \frac{d\eta}{dt} + 2k[\sigma_n^2 I + \Lambda_s]\eta = 2k[P + Q] \quad .$$

Making use of Eqs. (60) and (98) yields for the four η_i equations,

$$(101) \qquad \frac{d\eta_1}{dt} + 2k\sigma_n^2\eta_1 = 2kq_1 \quad ,$$

$$(102) \qquad \frac{d\eta_2}{dt} + 2k\sigma_n^2\eta_2 = 2kq_2 \quad ,$$

$$(103) \qquad \frac{d\eta_3}{dt} + 2k(\sigma_n^2+a^2)\eta_3 = 2k \left[\frac{a}{\sqrt{2}} + q_3\right] \quad ,$$

and

$$(104) \qquad \frac{d\eta_4}{dt} + 2k(\sigma_n^2+a^2)\eta_4 = 2kq_4 \quad .$$

The solutions are

$$(105) \qquad \eta_1(t) = \left[\eta_1(0) - \frac{q_1}{\sigma_n^2}\right] e^{-2k\sigma_n^2 t} + \frac{q_1}{\sigma_n^2} \quad ,$$

$$(106) \qquad \eta_2(t) = \left[\eta_2(0) - \frac{q_2}{\sigma_n^2}\right] e^{-2k\sigma_n^2 t} + \frac{q_2}{\sigma_n^2}$$

$$(107) \qquad \eta_3(t) = \left[\eta_3(0) - \frac{q_3+a/\sqrt{2}}{\sigma_n^2+a^2}\right] e^{-2k(\sigma_n^2+a^2)t} + \frac{q_3+a/\sqrt{2}}{\sigma_n^2+a^2} \quad ,$$

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and

$$(108) \quad n_4(t) = \left[n_4(0) - \frac{q_4}{\sigma_n^2 + a^2} \right] e^{-2k(\sigma_n^2 + a^2)t} + \frac{q_4}{\sigma_n^2 + a^2} \quad .$$

Assuming the same initial conditions as before (see Eqs. (70) and (71)), we find

$$(109) \quad n_1(t) = \left[-\frac{\cos\phi_1}{\sqrt{2}} - \frac{q_1}{\sigma_n^2} \right] e^{-2k\sigma_n^2 t} + \frac{q_1}{\sigma_n^2} \quad ,$$

$$(110) \quad n_2(t) = \left[-\frac{\sin\phi_1}{\sqrt{2}} - \frac{q_2}{\sigma_n^2} \right] e^{-2k\sigma_n^2 t} + \frac{q_2}{\sigma_n^2} \quad ,$$

$$(111) \quad n_3(t) = \left[\frac{\cos\phi_1}{\sqrt{2}} - \frac{q_3 + a/\sqrt{2}}{\sigma_n^2 + a^2} \right] e^{-2k(\sigma_n^2 + a^2)t} + \frac{q_3 + a/\sqrt{2}}{\sigma_n^2 + a^2} \quad ,$$

and

$$(112) \quad n_4(t) = \left[\frac{\sin\phi_1}{\sqrt{2}} - \frac{q_4}{\sigma_n^2 + a^2} \right] e^{-2k(\sigma_n^2 + a^2)t} + \frac{q_4}{\sigma_n^2 + a^2} \quad .$$

Finally, using Eq. (14) to calculate the w_i's (and using Eq. (20) to replace the q_i with the δ_i), we obtain for the complete solution for the w_i's,

$$(113) \quad w_1(t) = \left[\frac{1}{2} + \frac{1}{2\sigma_n^2} (-\delta_1 + \delta_3 \cos\phi_1 + \delta_4 \sin\phi_1) \right] e^{-2k\sigma_n^2 t}$$

$$+ \left[\frac{1}{2} - \frac{a \cos\phi_1}{2(\sigma_n^2 + a^2)} - \frac{1}{2(\sigma_n^2 + a^2)} (\delta_1 + \delta_3 \cos\phi_1 + \delta_4 \sin\phi_1) \right]$$

$$e^{-2k(\sigma_n^2 + a^2)t}$$

$$- \frac{1}{2\sigma_n^2} (-\delta_1 + \delta_3 \cos\phi_1 + \delta_4 \sin\phi_1) + \frac{a \cos\phi_1}{2(\sigma_n^2 + a^2)}$$

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(113)
cont.
$$+ \frac{1}{2(\sigma_n^2 + a^2)} (\delta_1 + \delta_3 \cos\phi_1 + \delta_4 \sin\phi_1) \quad ,$$

(114)
$$w_2(t) = - \frac{1}{2\sigma_n^2} (\delta_2 + \delta_3 \sin\phi_1 - \delta_4 \cos\phi_1) e^{-2k\sigma_n^2 t}$$

$$+ \left[\frac{a \sin\phi_1}{2(\sigma_n^2 + a^2)} + \frac{1}{2(\sigma_n^2 + a^2)} (-\delta_2 + \delta_3 \sin\phi_1 - \delta_4 \cos\phi_1) \right] e^{-2k(\sigma_n^2 + a^2)t}$$

$$+ \frac{1}{2\sigma_n^2} (\delta_2 + \delta_3 \sin\phi_1 - \delta_4 \cos\phi_1) - \frac{a \sin\phi_1}{2(\sigma_n^2 + a^2)}$$

$$- \frac{1}{2(\sigma_n^2 + a^2)} (-\delta_2 + \delta_3 \sin\phi_1 - \delta_4 \cos\phi_1) \quad ,$$

(115)
$$w_3(t) = \left[- \frac{\cos\phi_1}{2} - \frac{1}{2\sigma_n^2} (-\delta_1 \cos\phi_1 + \delta_2 \sin\phi_1 + \delta_3) \right] e^{-2k\sigma_n^2 t}$$

$$+ \frac{1}{2\sigma_n^2} (-\delta_1 \cos\phi_1 + \delta_2 \sin\phi_1 + \delta_3)$$

$$+ \left[\frac{\cos\phi_1}{2} - \frac{a}{2(\sigma_n^2 + a^2)} - \frac{1}{2(\sigma_n^2 + a^2)} (\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1 + \delta_3) \right]$$
$$e^{-2k(\sigma_n^2 + a^2)t}$$

$$+ \frac{a}{2(\sigma_n^2 + a^2)} + \frac{1}{2(\sigma_n^2 + a^2)} (\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1 + \delta_3)$$

and

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$$(116) \quad w_4(t) = \left[-\frac{\sin\phi_1}{2} - \frac{1}{2\sigma_n{}^2} (-\delta_1 \sin\phi_1 - \delta_2 \cos\phi_1 + \delta_4) \right] e^{-2k\sigma_n{}^2 t}$$

$$+ \frac{1}{2\sigma_n{}^2} (-\delta_1 \sin\phi_1 - \delta_2 \cos\phi_1 + \delta_4) +$$

$$+ \left[\frac{\sin\phi_1}{2} - \frac{1}{2(\sigma_n{}^2 + a^2)} (\delta_1 \sin\phi_1 + \delta_2 \cos\phi_1 + \delta_4) \right] e^{-2k(\sigma_n{}^2 + a^2) t}$$

$$+ \frac{1}{2(\sigma_n{}^2 + a^2)} (\delta_1 \sin\phi_1 + \delta_2 \cos\phi_1 + \delta_4) \quad .$$

The effect of the noise and the offset voltages can now be seen.

First, suppose there are no offset voltages present. Then the weights become

$$(117) \quad w_1(t) = \frac{1}{2} e^{-2k\sigma_n{}^2 t} + \left[\frac{1}{2} - \frac{a \cos\phi_1}{2(\sigma_n{}^2 + a^2)} \right] e^{-2k(\sigma_n{}^2 + a^2) t}$$

$$+ \frac{a \cos\phi_1}{2(\sigma_n{}^2 + a^2)}$$

$$(118) \quad w_2(t) = \frac{a \sin\phi_1}{2(\sigma_n{}^2 + a^2)} e^{-2k(\sigma_n{}^2 + a^2) t} - \frac{a \sin\phi_1}{2(\sigma_n{}^2 - a^2)} \quad ,$$

$$(119) \quad w_3(t) = -\frac{\cos\phi_1}{2} e^{-2k\sigma_n{}^2 t} + \left[\frac{\cos\phi_1}{2} - \frac{a}{2(\sigma_n{}^2 + a^2)} \right] e^{-2k(\sigma_n{}^2 + a^2) t}$$

$$+ \frac{a}{2(\sigma_n{}^2 + a^2)} \quad ,$$

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and

$$(120) \qquad w_4(t) = -\frac{\sin\phi_1}{2} e^{-2k\sigma_n^2 t} + \frac{\sin\phi_1}{2} e^{-2k(\sigma_n^2 + a^2)t} \qquad .$$

The final values of these weights are given by

$$(121) \qquad w_1(\infty) = \frac{a\cos\phi_1}{2(\sigma_n^2 + a^2)}$$

$$(122) \qquad w_2(\infty) = -\frac{a\sin\phi_1}{2(\sigma_n^2 + a^2)}$$

$$(123) \qquad w_3(\infty) = \frac{a}{2(\sigma_n^2 + a^2)}$$

$$(124) \qquad w_4(\infty) = 0$$

The desired part of the array output is then

$$(125) \qquad s(t) = \sum_{i=1}^{4} w_i(\infty)s_i(t) = \frac{a\cos\phi_1}{2(\sigma_n^2 + a^2)} [a\cos(\omega_1 t - \phi_1)]$$

$$- \frac{a\sin\phi_1}{2(\sigma_n^2 + a^2)} [a\sin(\omega_1 t - \phi_1)] + \frac{a}{2(\sigma_n^2 + a^2)} [a\cos(\omega_1 t)]$$

$$= \frac{a^2}{\sigma_n^2 + a^2} \cos\omega_1 t \qquad .$$

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It is interesting that because of the noise, the array output does not match the reference signal exactly. The amplitudes differ by the factor $a^2/\sigma_n{}^2 + a^2$. Although this is almost unity when the noise is small, it can be substantially less than unity if $\sigma_n{}^2$ is large. Basically, the array makes a compromise between the contributions to $\varepsilon^2(t)$ due to the noise and those due to the signal. A weight setting that would cause the desired output to match the reference signal exactly would result in a larger total mean-square error, because of the noise.

It is also interesting to compute the signal-to-noise ratio at the array output and in the error signal. Since the noise signals on each channel are incoherent, the total noise power in the array output, N_0, is the sum of the noise powers from each element:

$$(126) \qquad N_0 = \sigma_n{}^2 \sum_{i=1}^{4} w_1{}^2(\infty)$$

$$= \frac{a^2 \sigma_n{}^2}{2(\sigma_n{}^2 + a^2)^2} \qquad .$$

The signal power at the array output is

$$(127) \qquad S_0 = \frac{1}{2} \frac{a^4}{(\sigma_n{}^2 + a^2)^2} \qquad .$$

Hence the signal-to-noise ratio (SNR) is

$$(128) \qquad \frac{S_0}{N_0} = \frac{a^2}{\sigma_n{}^2} \qquad .$$

This is the maximum SNR it is possible to obtain with this array and the given signals.* Hence we see that minimum $\varepsilon^2(t)$ corresponds to

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*The maximum SNR that can be obtained for an array of N elements is $\sum_{i=1}^{N} (SNR)_i$, where $(SNR)_i$ is the SNR on the i-th element (see Reference 13, Eq. (25)). The SNR on each element in this example is $a^2/2\sigma_n{}^2$.

maximum SNR, and in the absence of offset voltages, this condition is achieved by the array. Also, we observe that the desired signal portion of the error signal, $\varepsilon_s(t)$, is

$$(129) \qquad \varepsilon_s(t) = R(t) - \sum_{i=1}^{4} w_i(\infty)s_i(t)$$

$$= \cos\omega_1 t - \frac{a^2}{\sigma_n^2 + a^2} \cos\omega_1 t$$

$$= \frac{\sigma_n^2}{\sigma_n^2 + a^2} \cos\omega_1 t \qquad .$$

Hence the desired signal power in the error signal is

$$(130) \qquad \overline{\varepsilon_s^2(t)} = \frac{1}{2} \frac{\sigma_n^4}{(\sigma_n^2 + a^2)^2} \qquad .$$

Since the noise power in the error signal, $\overline{\varepsilon_n^2(t)}$, is the same as that in the array output, the SNR in the error signal is

$$(131) \qquad \frac{\overline{\varepsilon_s^2(t)}}{\overline{\varepsilon_n^2(t)}} = \frac{\sigma_n^2}{a^2} \qquad ,$$

the reciprocal of the SNR at the array output. Similar SNR inversion effects have been noted previously by Zahm[14].

Now we examine the weights when offset voltages are present. From Eqs. (113) - (116), the steady-state weights in this case will be given by

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(132) $\quad w_1(\infty) = \dfrac{a \cos\phi_1}{2(\sigma_n^2 + a^2)} + \dfrac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ \delta_1$

$$- \dfrac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ (\delta_3 \cos\phi_1 + \delta_4 \sin\phi_1) \qquad ,$$

(133) $\quad w_2(\infty) = -\dfrac{a \sin\phi_1}{2(\sigma_n^2 + a^2)} + \dfrac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ \delta_2$

$$+ \dfrac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ (\delta_3 \sin\phi_1 - \delta_4 \cos\phi_1) \qquad ,$$

(134) $\quad w_3(\infty) = \dfrac{a}{2(\sigma_n^2 + a^2)} - \dfrac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ (\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1)$

$$+ \dfrac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ \delta_3$$

and

(135) $\quad w_4(\infty) = -\dfrac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ (\delta_1 \sin\phi_1 + \delta_2 \cos\phi_1) + \dfrac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)}\ \delta_4 \quad .$

The last two terms in each of these expressions represent the change in the weight, away from its optimum point, because of the offset voltages. Whether this change is significant depends, of course, on the values of the various coefficients. Clearly, both

(136) $\quad \lim\limits_{\sigma_n^2 \to 0}\ \dfrac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)} = +\infty \qquad ,$

and

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$$(137) \qquad \lim_{\sigma_n^2 \to 0} \frac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)} = +\infty \quad ,$$

whereas the coefficient $a/2(\sigma_n^2 + a^2)$ is bounded,

$$(138) \qquad \lim_{\sigma_n^2 \to 0} \frac{a}{2(\sigma_n^2 + a^2)} = \frac{1}{2a} \quad .$$

Thus, the offset voltage terms will dominate the final weights if σ_n^2 is small enough. When the final weights are different than the values in Eqs. (121) - (124), the SNR at the array output will obviously be poorer than in Eq. (128).

To obtain good performance from the array, it will be necessary to keep the last two terms in Eqs. (132) - (135) small. If the values of σ_n^2 and a are given, the multiplier circuits must perform so that the δ_i are small enough that they have only a minor effect on the final weight setting.

In practice, however, the opposite problem is sometimes true. After the array is built, it is found that the offset voltage can be held only to within certain values. One is then faced with certain offset voltages, and the problem is to choose the noise level in the array so good performance results. Consider, for example, the problem of keeping the last two terms in Eq. (132) small compared with the first term. Suppose the offset voltages can be held only to within a value D:

$$(139) \qquad |\delta_i| \leq D \quad \text{for all } i \quad .$$

(Note that the quantity $\delta_3 \cos\phi_1 + \delta_4 \sin\phi_1$ is just a coordinate rotation in the offsets δ_3 and δ_4, so we also assume $|\delta_3 \cos\phi_1 + \delta_4 \sin\phi_1| \leq D$.) Therefore the last two terms of Eq. (132) are bounded by the quantity

$$(140) \qquad \frac{2\sigma_n^2 + a^2}{2\sigma_n^2(\sigma_n^2 + a^2)} D + \frac{a^2}{2\sigma_n^2(\sigma_n^2 + a^2)} D = \frac{\sigma_n^2 + a^2}{\sigma_n^2(\sigma_n^2 + a^2)} D \quad .$$

To keep this small compared to the first term in Eq. (132), we must keep the ratio

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$$
(141) \qquad \frac{\dfrac{\sigma_n^2 + a^2}{\sigma_n^2(\sigma_n^2 + a^2)} D}{\dfrac{a \cos \phi_1}{2(\sigma_n^2 + a^2)}} = \frac{\sigma_n^2 + a^2}{\sigma_n^2} \quad \frac{2D}{a \cos \phi_1}
$$

small. This ratio is shown plotted versus σ_n^2 in Fig. 4.
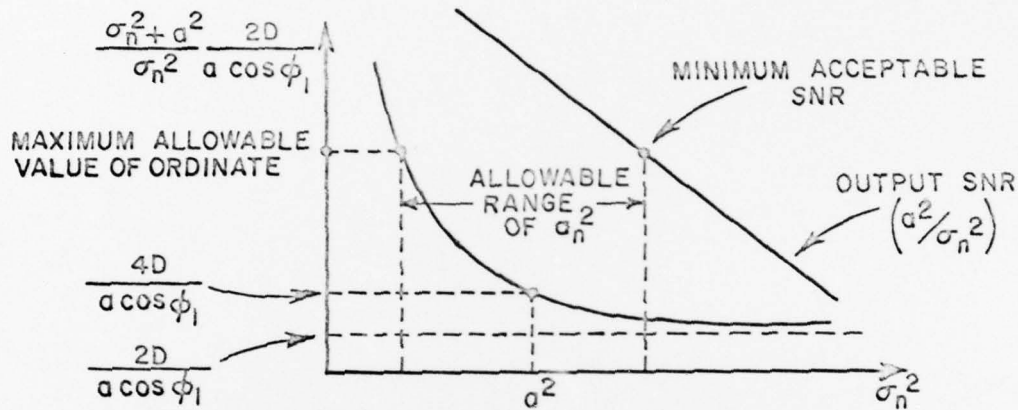

Fig. 4. Tradeoff between offset voltages and output SNR.

It is seen that keeping the offset terms small will require the noise to be larger than some minimum amount. On the other hand, as σ_n^2 increases, the SNR at the array output (a^2/σ_n^2) drops. Thus, σ_n^2 must be chosen to compromise between these conflicting requirements. There is a finite range of values for σ_n^2 where suitable array performance is obtained. This fact has frequently been observed experimentally (see Reference 6, page 8). In general, the larger the value of D, the smaller will be the allowed range of σ_n^2. If D is high enough, values of σ_n^2 yielding good weight values will result in too low an SNR for the communication system.

Now let us compute the SNR at the array output when offset voltages are present. The desired signal output is given by

$$
(142) \qquad \text{Desired Signal Output} = \sum_{i=1}^{4} w_i(\infty) s_i(t) \quad .
$$

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Using Eqs. (48) and (132) - (135) to evauate this yields

$$(143) \quad \sum_{i=1}^{4} w_i(\infty)s_i(t) = \frac{a^2}{\sigma_n^2 + a^2} \left\{ \left[1 + \frac{1}{a} (\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1 + \delta_3) \right] \cos\omega_1 t \right.$$

$$\left. + \left[\frac{1}{a} (\delta_1 \sin\phi_1 + \delta_2 \cos\phi_1 + \delta_4) \right] \sin\omega_1 t \right\} .$$

We see that not only is the amplitude of the desired signal changed by the offsets, but also a quadrature component ($\sin\omega_1 t$) now appears at the output as well.* The total signal power at the array output may be computed from Eq. (143), with the result

------

*This fact has certain implications for adaptive array systems in which the reference signal is generated from the array output in a bootstrap loop[7,8,9]. In such systems, the phase shift around the reference signal generation loop is normally adjusted so the reference signal is in phase with the desired signal at the array output. In the presence of offset voltages, the array may be seeking a steady-state condition in which the array output is not in phase with the reference signal. Thus, as the reference signal phase tracks the phase of the array output, a cycling of the weights may result.

$$(144) \quad S_o = \frac{a^4}{2(\sigma_n{}^2+a^2)^2} \left\{ \left[1 + \frac{1}{a}(\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1 + \delta_3) \right]^2 \right.$$

$$\left. + \left[\frac{1}{a}(\delta_1 \sin\phi_1 + \delta_2 \cos\phi_1 + \delta_4) \right]^2 \right\}$$

$$= \frac{a^4}{2(\sigma_n{}^2+a^2)} \left\{ 1 + \frac{2}{a}(\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1 + \delta_3) \right.$$

$$+ \frac{1}{a^2}(\delta_1{}^2+\delta_2{}^2+\delta_3{}^2+\delta_4{}^2) + \frac{2}{a^2}(\delta_1\delta_3 \cos\phi_1 - \delta_2\delta_3 \sin\phi_1$$

$$\left. + \delta_1\delta_4 \sin\phi_1 + \delta_2\delta_4 \cos\phi_1) \right\} \quad .$$

Similarly, the total noise power at the array output is found to be:

$$(145) \quad N_o = \sigma_n{}^2 \sum_{i=1}^{4} w_i{}^2(\infty) = \frac{1}{2\sigma_n{}^2(\sigma_n{}^2+a^2)} \left\{ a^2\sigma_n{}^4 + 2a\sigma_n{}^4(\delta_1 \cos\phi_1 - \delta_2 \sin\phi_1) \right.$$

$$+ (2\sigma_n{}^2+2a^2\sigma_n{}^2+a^4)(\delta_1{}^2+\delta_2{}^2+\delta_3{}^2+\delta_4{}^2) - 2a^2(2\sigma_n{}^2+a^2)$$

$$(\delta_1\delta_3 \cos\phi_1 + \delta_1\delta_4 \sin\phi_1 - \delta_2\delta_3 \sin\phi_1 + \delta_2\delta_4 \cos\phi_1)$$

$$\left. + 2a\sigma_n{}^4\delta_3 \right\} \quad .$$

The SNR can be calculated from these two quantities for specific values of δ_i and ϕ_1. As an example, suppose $\delta_1 = \delta_2 = \delta_4 = 0$ and $\delta_3 \neq 0$. Then we find

$$(146) \quad S_o = \frac{a^4}{2(\sigma_n{}^2+a^2)} \left\{ 1 + \frac{2\delta_3}{a} + \frac{\delta_3{}^2}{a^2} \right\} = \frac{a^4}{2(\sigma_n{}^2+a^2)} \left(1 + \frac{\delta_3}{a} \right)^2 \quad ,$$

and

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$$(147) \quad N_0 = \frac{1}{2\sigma_n^2(\sigma_n^2+a^2)} \left\{ a^2\sigma_n^4 + (2\sigma_n^4 + 2a^2\sigma_n^2 + a^4)\delta_3^2 \\ + 2a\sigma_n^4\delta_3 \right\} \quad .$$

Hence

$$(148) \quad \frac{S_0}{N_0} = \sigma_n^2 a^4 \left\{ \frac{\left(1 + \frac{\delta_3}{a}\right)^2}{a^2\sigma_n^4 + (2\sigma_n^4 + 2a^2\sigma_n^2 + a^4)\delta_3^2 + 2a\sigma_n^4\delta_3} \right\}$$

$$= \frac{\frac{a^2}{\sigma_n^2}}{1 + \left(1 + \frac{a^2}{\sigma_n^2}\right)^2 \left[\frac{\frac{\delta_3}{a}}{1 + \frac{\delta_3}{a}}\right]^2} \quad .$$

This SNR is smaller than the SNR in the absence of offsets, as expected. It is interesting that for a fixed offset voltage δ_3, there is a maximum SNR that can be obtained. Differentiating Eq. (148) with respect to the quantity a^2/σ_n^2 shows that S_0/N_0 is maximum when

$$(149) \quad \frac{a^2}{\sigma_n^2} = \frac{a}{\delta_3} \sqrt{1 + 2\left(\frac{\delta_3}{a}\right) + 2\left(\frac{\delta_3}{a}\right)^2} \quad .$$

For this value of a^2/σ_n^2, S_0/N_0 is found to be

$$(150) \quad \frac{S_0}{N_0} = \frac{1 + 2\left(\frac{\delta_3}{a}\right) + \left(\frac{\delta_3}{a}\right)^2}{2\left(\frac{\delta_3}{a}\right)\left[\sqrt{1 + 2\left(\frac{\delta_3}{a}\right) + \left(\frac{\delta_3}{a}\right)^2} + 2\left(\frac{\delta_3}{a}\right)\right]} \quad .$$

Larger or smaller values of a^2/σ_n^2 than that in Eq. (149) yield lower overall output SNR. The maximum output SNR is larger, of course, the smaller δ_3/a.

V.   PRACTICAL SYSTEM LIMITATIONS

In addition to the effect of offsets on the final weights and the array performance, discussed above, there is another practical aspect of the problem. In a real system the array weights have only a finite linear operating range. Each weight is the output of an integrator, usually implemented with an operational amplifier. The output of this integrator will have only a certain range over which it can vary. Thus, if the theoretical solution for the weight behavior indicates that the weight should increase to a large value, it may not be capable of achieving that value because of equipment limitations.

Consider again the equations for the normal weights $n_i(t)$. Some of these, in particular those associated with zero eigenvalues of ϕ_s (for which the eigenvalue of ϕ_{s+n} is σ_n^2), satisfy the differential Eq. (26),

$$(151) \qquad \frac{dn_i}{dt} + 2k\sigma_n^2 n_i = 2k(p_i + q_i) \qquad ,$$

whose solution is given by Eqs. (27) and (28),

$$(152) \qquad n_i(t) = \left[n_i(0) - \frac{p_i + q_i}{\sigma_n^2} \right] e^{-2k\sigma_n^2 t} + \frac{p_i + q_i}{\sigma_n^2} \qquad .$$

This is a transient starting at $n_i(0)$ and ending at a final value of

$$(153) \qquad n_i(\infty) = \frac{p_i + q_i}{\sigma_n^2} \qquad ,$$

as shown in Fig. 5.

For nonzero p_i or q_i, this final value becomes arbitrarily large as $\sigma_n^2 \to 0$, so it can easily dominate the steady-state solution for the w_i. If σ_n^2 is small enough, these terms drive the w_i to the limits of their operating range.
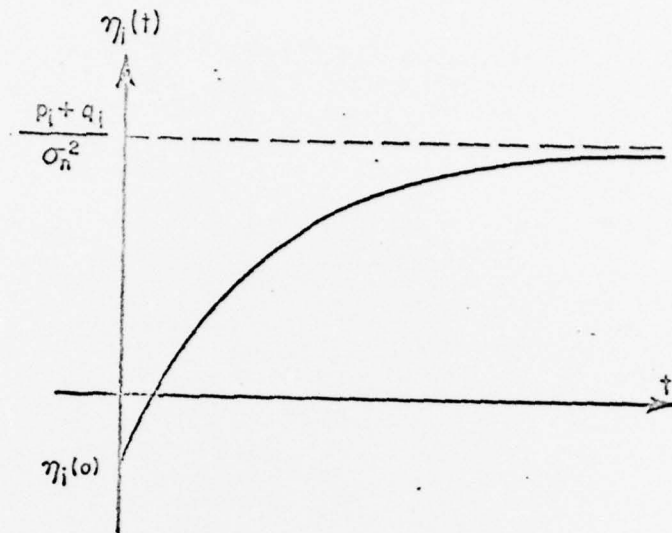
Fig. 5. Transient response of $n_i(t)$.

For example, consider our example of the two-element array with one signal incident and element noise present. The final values for the weights were given in Eqs. (132) - (135). We see that for small σ_n^2, the final values of the weights are given approximately by

(154) $\qquad w_1(\infty) \overset{\sim}{=} \dfrac{1}{2\sigma_n^2} (\delta_1 - \delta_3 \cos\phi_1 - \delta_4 \sin\phi_1)$

(155) $\qquad w_2(\infty) \overset{\sim}{=} \dfrac{1}{2\sigma_n^2} (\delta_2 + \delta_3 \sin\phi_1 - \delta_4 \cos\phi_1)$

(156) $\qquad w_3(\infty) \overset{\sim}{=} \dfrac{1}{2\sigma_n^2} (\delta_3 - \delta_1 \cos\phi_1 + \delta_2 \sin\phi_1)$

(157) $\qquad w_4(\infty) \overset{\sim}{=} \dfrac{1}{2\sigma_n^2} (\delta_4 - \delta_1 \sin\phi_1 - \delta_2 \cos\phi_1)$.

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As long as the offsets δ_i are not zero, these weights can be arbitrarily large for small σ_n^2. Clearly, the larger the δ_i, the larger σ_n^2 will have to be to keep each of the weights within its linear operating range.

When one of the weights hits its saturation limit during an adaptation transient, the value of that weight remains constant from then on. The behavior of the system after this time can be found by setting up a new system of differential equations for the remaining weights. If the array is noise-free and the Φ matrix is singular, fixing one of the weights will reduce the order of the system to $2N-1$ without reducing the rank of the (new) Φ matrix for the remaining $2N-1$ weights. In general, as many weights will go into saturation as are required to reduce the number of remaining variable weights to the rank of Φ. Expressed another way, each weight going into saturation uses up one of the unneeded degrees of freedom in the antenna pattern. As many will go into saturation as there are extra degrees of freedom.

If element noise is present, so Φ is nonsingular, the final weight vector obtained if one or more weights go into saturation will no longer yield minimum error signal. Since there are no extra degrees of freedom when noise is present, there is a unique weight vector yielding minimum $\epsilon^2(t)$. If this weight vector lies outside the linear operating range for some of the weights, it cannot be attained by the array. When certain of the weights are constrained by saturation, the remaining weights will go to the values yielding minimum $\overline{\epsilon^2(t)}$ subject to the weight constraints. This value of $\epsilon^2(t)$ will not be as low, however, as could have been obtained without the constraints.

VI. CONCLUSIONS

The effects of multiplier offset voltages have been studied. The offset voltages δ_i enter the differential equations for the array weights as shown in Eqs. (8) or (13). These equations may be solved by making a coordinate rotation of the weights, as shown in Eq. (14). The resulting differential equations, Eq. (21), can be solved for the normal weights $n_i(t)$ versus time. When the Φ matrix in Eq. (9) is singular (which happens only when the array is noise-free and there are fewer signals incident than there are elements), certain of the $n_i(t)$ exhibit ramp solutions, as given in Eq. (31). In this case, the weights w_i rise to arbitrarily high values. For the noisy case, all $n_i(t)$ have decaying exponential solutions of the type in Eq. (27). In this case, the weights w_i can also rise to very large final values if offset voltages are present and there is insufficient noise.

A simple example was given in Section IV to illustrate the application of these results to a specific case -- a 2-element array with one signal incident and noise present. The effects of the offsets may

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be seen in the resulting weight behavior. For given offsets, there is a minimum amount of noise for which the effect of the offsets is negligible. As the noise is reduced below this value, the offset terms dominate the weight solutions. The output SNR from the array, which would be optimum in the absence of the offsets, is found to be degraded because of the offsets. Furthermore, it is shown that with offset voltages present, there is a maximum output SNR that can be achieved. Input noise levels that are either too high or too low result in a poorer output SNR from the array.

Finally, it is shown that the finite operating range of the weights in real equipment imposes a further constraint on the acceptable values of the offset voltages and the noise.

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Technology Service Corporation
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Radar Microwave Laboratory
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1. ORIGINATING ACTIVITY *(Corporate author)* ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University, Columbus, Ohio 43212	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE

MULTIPLIER OFFSET VOLTAGES IN ADAPTIVE ARRAYS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Quarterly Report

5. AUTHOR(S) *(Last name, first name, initial)*

Compton, R.T., Jr.

6. REPORT DATE October 1972	7a. TOTAL NO. OF PAGES 44	7b. NO. OF REFS 14
8a. CONTRACT OR GRANT NO. Contract N00019-72-C-0184	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.	ElectroScience Laboratory 3433-1	
c. TASK	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	
d.		

10. AVAILABILITY/LIMITATION NOTICES

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy Naval Air Systems Command Washington, D.C. 20360

13. ABSTRACT

This report examines the effects of multiplier offset voltages in adaptive arrays. Multiplier offset voltages arise when active circuits are used to implement the error-by-signal multipliers required in an array based on the LMS algorithm. These offset voltages are known from experimental work to have a strong effect on array performance.

It is first shown how multiplier offset voltages may be included in the differential equations for the array weights. Then their effect on weight behavior is studied. It is found that the offset voltages affect the final values of the weights, but not the time constants. Furthermore, the effect they have is influenced by the amount of element noise in the array. Ad adequate amount of noise is necessary to minimize weight errors due to offset voltages.

An example is treated to show the effect of offset voltages on the final array weights and the output SNR. With offset voltages present, it is found that there is a maximum SNR that can be obtained from the array. A specific input SNR is required to obtain this maximum output SNR.

Finally, it is shown that a finite operating range for the weights places a further restriction on the acceptable values of offset voltages and noise.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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